LOCAL HEATING OF AN ANISOTROPIC BODY BY A CIRCULAR HEAT SOURCE

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The solution and analysis of the two-dimensional nonstationary heat-conduction problem are presented for an anisotropic (orthotropic) body heated by a circular heat source.

Kozlov [1] reported on an investigation of three-dimensional nonstationary temperature fields originating in isotropic bodies heated by bounded heat sources of different geometric shape. Let us consider a semibounded (in thermal respects) anisotropic (orthotropic) body to whose surface a specific heat flux of density $q(\tau)$ arbitrary in time is delivered through a given circular domain $0 \le r \le r_0$ $(x = 0)$. Ideal heat insulation exists on the remaining part of the surface $x = 0$, $r > r_0$. In this problem cylindrical anisotropy is examined, i.e., only in the directions of the cylindrical r, x coordinates are the heat conductivity (λ_r and λ_x), the thermal diffusivity (a_r, a_x), and the thermal activity (b_r and b_x) different from each other. We consider the bulk specific heats equal in the corresponding directions, i.e., $c_r\gamma_r = c_x\gamma_x$. Therefore

$$K_\lambda = \frac{\lambda_r}{\lambda_x} = \frac{a_r}{a_x} = K_a = K_b^2 = b_r^2/b_x^2 . \tag{1}$$

MATHEMATICAL FORMULATION OF THE PROBLEM

In conformity with the formulation of this problem, it is required to solve two differential heat-conduction equations of the form

$$\begin{aligned} &\frac{a_r}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Theta_1}{\partial r}\right) + a_x\frac{\partial^2\Theta_1}{\partial x^2} = \frac{\partial\Theta_1}{\partial\tau}\ (r < r_0,\ x \geqslant 0,\ \tau > 0),\\ &\frac{a_r}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\Theta_2}{\partial r}\right) + a_x\frac{\partial^2\Theta_2}{\partial x^2} = \frac{\partial\Theta_2}{\partial\tau}\ (r > r_0 > 0,\ x \geqslant 0,\ \tau > 0) \end{aligned} \tag{2}$$

under the initial conditions

$$T_1(r,\ x,\ 0) - T_0 = \Theta_1(r,\ x,\ 0) = \Theta_2(r,\ x,\ 0) = T_2(r,\ x,\ 0) - T_0 = 0 \tag{3}$$

and the boundary conditions

$$\begin{aligned} &\frac{\partial\Theta_1(r,\ 0,\ \tau)}{\partial x} = -\frac{q(\tau)}{\lambda_x}\ (0 \leqslant r < r_0,\ \tau > 0,\ x = 0),\\ &\frac{\partial\Theta_2(r,\ 0,\ \tau)}{\partial x} = 0\ (r > r_0,\ \tau > 0),\quad \frac{\partial\Theta_1(0,\ x,\ \tau)}{\partial r} = 0\ (x \geqslant 0),\\ &\frac{\partial\Theta_1(r,\ \infty,\ \tau)}{\partial x} = \frac{\partial\Theta_2(r,\ \infty,\ \tau)}{\partial x} = \frac{\partial\Theta_2(\infty,\ x,\ \tau)}{\partial r} = 0,\\ &\frac{\partial\Theta_1(r_0,\ x,\ \tau)}{\partial r} = \frac{\partial\Theta_2(r_0,\ x,\ \tau)}{\partial r},\ \Theta_1(r_0,\ x,\ \tau) = \Theta_2(r_0,\ x,\ \tau). \end{aligned} \tag{4}$$

The general solution of this problem to determine the excess temperatures

$$\Theta_1(r,\ x,\ \tau) = T_1(r,\ x,\ \tau) - T_0 \text{ for } 0 \leqslant r \leqslant r_0,\ x \geqslant 0,\ \tau > 0,$$

$$\Theta_2(r,\ x,\ \tau) = T_2(r,\ x,\ \tau) - T_0 \text{ for } r > r_0,\ x \geqslant 0,\ \tau > 0,$$

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obtained by operator methods can be represented in the following form:

$$\Theta_1(r,\ x,\ \tau) = \frac{1}{b_x \sqrt{\pi}} \int_0^{\tau} \exp\left[-\frac{x^2}{4a_x(\tau-\xi)}\right] \frac{q(\xi)}{\sqrt{\tau-\xi}}\, d\xi -$$

$$-\frac{r_0}{\lambda_x} \frac{K_a^{-1/2}\sqrt{x}}{i\sqrt{2\pi^3}} \int_{\sigma-i\infty}^{\sigma+i\infty} \int_0^{\infty} \exp(s\tau) \frac{J_{-1/2}(px)\sqrt{p}}{\left(p^2+\frac{s}{a_x}\right)^{1/2}} \times \tag{5}$$

$$\times K_1\left(r_0 \sqrt{K_a^{-1}\left(p^2+\frac{s}{a_x}\right)}\right) I_0\left(r \sqrt{K_a^{-1}\left(p^2+\frac{s}{a_x}\right)}\right) q(s)\, dpds,$$

$$\Theta_2(r,\ x,\ \tau) = \frac{r_0}{\lambda_x} \frac{K_a^{-1/2}\sqrt{x}}{i\sqrt{2\pi^3}} \int_{\sigma-i\infty}^{\sigma+i\infty} \int_0^{\infty} \frac{\exp(s\tau)\, J_{-1/2}(px)\sqrt{p}}{(p^2+s/a_x)^{1/2}} \times \tag{6}$$

$$\times I_1\left(r_0 \sqrt{K_a^{-1}(p^2+s/a_x)}\right) K_0\left(r \sqrt{K_a^{-1}\left(p^2+\frac{s}{a_x}\right)}\right) q(s)\, dpds.$$

The solution on the axes r = 0, x ≥ 0, τ > 0 of an orthotropic body is expressed in the form of the following quadrature:

$$\Theta_1(0,\ x,\ \tau) = \frac{1}{b_x\sqrt{\pi}} \int_0^{\tau} \exp\left[\frac{-x^2}{4a_x(\tau-\xi)}\right]\left\{1-\exp\left[-\frac{K_a^{-1} r_0^2}{4a_x(\tau-\xi)}\right]\right\} \frac{q(\xi)\, d\xi}{\sqrt{\tau-\xi}}. \tag{7}$$

If the thermal diffusivity coefficients are $a_x = a_r$, then we have an analogous quadrature for an isotropic body [1].

For x = 0 the corresponding temperature values at any point of the orthotropic body surface are expressed in the form

$$\Theta_1(r,\ 0,\ \tau) = \frac{1}{b_x\sqrt{\pi}} \int_0^{\tau} \frac{q(\tau-\xi)}{\sqrt{\xi}}\, d\xi - \frac{1}{b_x}\sqrt{\frac{2}{\pi}} \times$$

$$\times \sum_{n=0}^{\infty} \sum_{m=0}^{n} A_{n,m} \left(\frac{r}{r_0}\right)^{2n} \left(\frac{r_0}{\sqrt{a_r}}\right)^{2n-m-\frac{1}{2}} \int_0^{\tau} q(\tau-\xi)\, \xi^{\frac{m}{2}-n-\frac{1}{4}} \times \tag{8}$$

$$\times \exp\left(-\frac{r_0^2}{8a_r\xi}\right) W_{n-\frac{m}{2}+\frac{1}{4},\ \frac{m}{2}-\frac{1}{4}}\left(\frac{r_0^2}{4a_r\xi}\right) d\xi \quad (0 \leqslant r < r_0),$$

$$\Theta_2(r,\ 0,\ \tau) = \frac{1}{b_x}\sqrt{\frac{2}{\pi}} \sum_{n=0}^{\infty} \sum_{m=0}^{n} \frac{A_{n,m}}{2(n+1)} \left(\frac{r_0}{r}\right)^{m+\frac{3}{2}} \times \tag{9}$$

$$\times \left(\frac{r_0}{\sqrt{a_r}}\right)^{2n-m+\frac{1}{2}} \int_0^{\tau} q(\tau-\xi)\, \xi^{-n+\frac{m}{2}-\frac{3}{4}} \exp\left(-\frac{r^2}{8a_r\xi}\right) W_{n-\frac{m}{2}+\frac{3}{4},\ \frac{m}{2}+\frac{1}{4}}\left(\frac{r^2}{4a_r\xi}\right) d\xi \quad (r > r_0),$$

where

$$A_{n,m} = \frac{C_n^m \left(\frac{1}{2}\right)_m 2^m}{4^n (n!)^2} = \frac{(2m-1)!!}{4^n n! m! (n-m)!}; \tag{10}$$

and $W_{k,\mu}(x)$ is the Whittaker function.

For certain practical applications associated with thermophysical measurements, it is expedient to present the expression for determination of the integrated surface temperature $\Theta_1(0 \le r \le r_0,\ 0,\ \tau) = T_1(0 \le r \le r_0,\ 0,\ \tau) - T_0$ in the domain of the circle $0 \le r \le r_0$ (x = 0):

$$\Theta_1(0\le r\le r_0,\ 0,\ \tau)=\frac{2}{r_0^2}\int_0^{r_0}\Theta_1(r,\ 0,\ \tau)\,r\,dr=\frac{1}{b_x\sqrt{\pi}}\int_0^{\tau}\frac{q(\tau-\xi)}{\sqrt{\xi}}d\xi-$$

$$-\frac{1}{b_x}\sqrt{\frac{2}{\pi}}\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{A_{n,m}}{n+1}\left(\frac{r_0}{\sqrt{a_r}}\right)^{2n-m-\frac{1}{2}}\int_0^{\tau}q(\tau-\xi)\,\xi^{\frac{m}{2}-n-\frac{1}{4}}\times \tag{11}$$

$$\times\exp\left(-\frac{r_0^2}{8a_r\xi}\right)W_{\frac{2n-m}{2}+\frac{1}{4},\ \frac{m}{2}-\frac{1}{4}}\left(\frac{r_0^2}{4a_r\xi}\right)d\xi.$$

Let us assume that $q(\tau - \xi) = q_0 = \text{const}$ in this domain ($0 \le r \le r_0$, $x = 0$) is independent of the time. Then by using (7)-(9), we find appropriate dependences to determine the temperature

$$\Theta_1^*(0,\ x,\ \tau)=\Theta_1(0,\ x,\ \tau)/T_0,\ \Theta_1^*(r,\ 0,\ \tau)=\Theta_1(r,\ 0,\ \tau)/T_0,$$

$$\Theta_2^*(r,\ 0,\ \tau)=\Theta_2(r,\ 0,\ \tau)/T_0, \tag{12}$$

$$\frac{\Theta_1^*(0,\ x,\ \tau)}{\mathrm{Ki}_x}=2\sqrt{\mathrm{Fo}_x}\left\{\mathrm{ierfc}\left(\frac{K_x}{2\sqrt{\mathrm{Fo}_x}}\right)-\right.$$

$$\left.-\,\mathrm{ierfc}\left(\frac{\sqrt{K_a^{-1}+K_x^2}}{2\sqrt{\mathrm{Fo}_x}}\right)\right\}\ (r=0,\ x\ge 0,\ \tau>0),$$

$$\frac{\Theta_1^*(r,\ 0,\ \tau)}{\mathrm{Ki}_x}=2\sqrt{\mathrm{Fo}_x}\left\{\frac{1}{\sqrt{\pi}}-\frac{1}{\sqrt{2\pi}}\exp\left(-\frac{K_a^{-1}}{8\,\mathrm{Fo}_x}\right)\times\right. \tag{13}$$

$$\left.\times(\mathrm{Fo}_xK_a)^{1/4}\sum_{n=0}^{\infty}\sum_{m=0}^{n}A_{n,m}K_r^{2n}\left(\frac{K_a^{-1}}{\mathrm{Fo}_x}\right)^{\frac{2n-m}{2}}W_{\frac{2n-m}{2}-\frac{3}{4},\ \frac{m}{2}-\frac{1}{4}}\left(\frac{K_a^{-1}}{4\,\mathrm{Fo}_x}\right)\right\}\ (x=0,\ 0\le r<r_0),$$

$$\frac{\Theta_2^*(r,\ 0,\ \tau)}{\mathrm{Ki}_x}=\frac{1}{\sqrt{2\pi}}\left(\frac{\mathrm{Fo}_x}{K_a}\right)^{1/4}K_r^{-3/2}\exp\left(-\frac{K_r^2}{8\,\mathrm{Fo}_xK_a}\right)\times \tag{14}$$

$$\times\sum_{n=0}^{\infty}\sum_{m=0}^{n}\frac{A_{n,m}}{(n+1)}K_r^{-m}\left(\frac{1}{\mathrm{Fo}_xK_a}\right)^{\frac{2n-m}{2}}W_{\frac{2n-m}{2}-\frac{1}{4},\ \frac{m}{2}+\frac{1}{4}}\left(\frac{K_r^2}{4\,\mathrm{Fo}_xK_a}\right),$$

$$x=0,\ r>r_0.$$

The dependence

$$\Theta_1^*(0,\ K_x,\ \mathrm{Fo}_x)/\mathrm{Ki}_x=\Theta_1^*/\mathrm{Ki}_x=\frac{T_1(0,\ x,\ \tau)-T_0}{q_0r_0/\lambda_x}$$

characterizes the change in the relative excess temperature on the axis $r = 0$, $x \ge 0$ of the orthotropic body under consideration heated by a circular heat source of constant intensity. These dependences are represented in Fig. 1. Presented in Fig. 1a are data for the central point of the heating spot ($x = r = 0$) as a function of the dimensionless time $\mathrm{Fo}_x = a_x\tau/r_0^2$ for fixed values of the parameter $K_a=a_r/a_x$. Analogous dependences for a specific point on the axis $r = 0$, $K_x = 0.5$ are presented in Fig. 1b. The deduction can be made from the represented graphs that at the initial times $\mathrm{Fo}_x < 0.1$ the temperature field development on the axis $r = 0$, $x \ge 0$ of the orthotropic body heated by a circular heat source of constant intensity is determined completely by the thermophysical properties in the direction of the x axis. The difference in the thermophysical properties in the direction of the r axis starts to exert influence on the nature of the temperature change at any point of the axis $r = 0$ for $\mathrm{Fo}_x > 0.1$. This deduction will later be used to determine the thermal activity b_x of the body under consideration at the initial times. For $\mathrm{Fo}_x > 0.1$ the parameter $K=a_r/a_x$ will exert considerable influence on the development of the temperature fields $\Theta_1^*(0, K_x, \mathrm{Fo}_x)/\mathrm{Ki}_x$. The functional influence of the parameter K_a on the formation of the temperature fields $\Theta_1^*(0, K_x, \mathrm{Fo}_x)/\mathrm{Ki}_x$ is seen from Fig. 1. This influence will be estimated by means of the relation to the dependence $\Theta_1^*(0, K_x, \mathrm{Fo}_x)/\mathrm{Ki}_x$ for $K_a = 1$, i.e., when $a_x = a_r$ ($\lambda_x = \lambda_r$ is the isotropic body case). The dependence $\Theta_1^*(0, K_x, \mathrm{Fo}_x)/\mathrm{Ki}_x$ in Fig. 1a corresponds for $K_a = 1$ to curve 9 and to curve 7 in Fig. 1b. All the dependences located above these

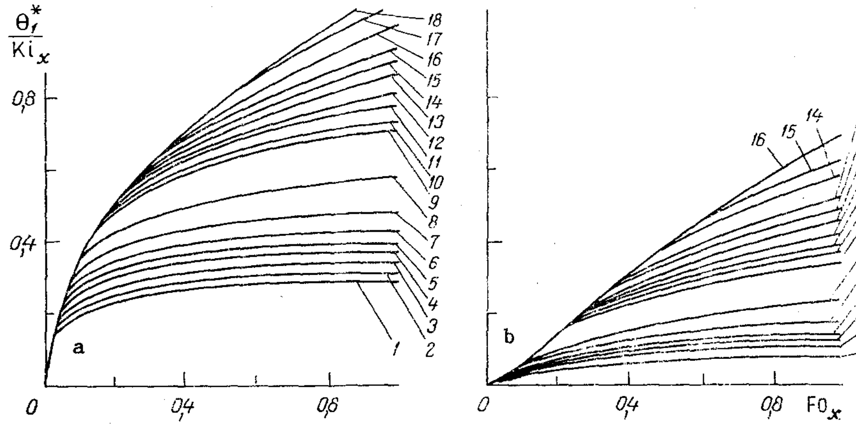

Fig. 1. Dependence of $\Theta_1^*(0, K_x, Fo_x)/Ki_x$ for different values of K_a: a) for $K_x = 0$: 1) $K_a = 10$; 2) 9; 3) 7; 4) 6; 5) 5; 6) 4; 7) 3; 8) 2; 9) 1; 10) 0.9; 11) 0.8; 12) 0.7; 13) 0.6; 14) 0.5; 15) 0.4; 16) 0.3; 17) 0.2; 18) $K_a = 0.1$; b) for $K_x = 0.5$: 1) $K_a = 7$; 2) 6; 3) 5; 4) 4; 5) 3; 6) 2; 7) 1.0; 8) 0.9; 9) 0.8; 10) 0.7; 11) 0.6; 12) 0.5; 13) 0.4; 14) 0.3; 15) 0.2; 16) 0.1.

curves ($k_a = 1$) will correspond to the value of the parameter $K_a<1$ ($a_r<a_x$), and the dependences located below to the parameter $K_a>1$ ($a_r>a_x$). The evident deduction follows that agrees well with the physical perception of the anisotropic body under consideration: for $K_a<1$ ($a_r<a_x$, $\lambda_r<\lambda_x$) the heat flux in the direction of the x axis predominates over the heat flux in the radial direction, and therefore the absolute level of the values of the temperature $\Theta_1^*(0, K_x, Fo_x)/Ki_x$ will be higher relative to the level of this temperature in an isotropic ($a_r = a_x$) body. If $a_r>a_x$ ($K_a>1$, $\lambda_r>\lambda_x$), then the heat flux in the radial direction will predominate over the axial (in the direction of the x axis) heat flux, and therefore, the absolute level of the temperature $\Theta_1^*(0, K_x, Fo_x)/Ki_x$ will be below the corresponding temperature level for $K_a = 1$.

If the limit cases of the change in the parameter K_a are considered then we obtain:

For small values of K_a ($K_a \to 0$)

$$\lim_{K_a\to 0} \frac{\Theta_1^*(0,\ 0,\ \tau)}{\mathrm{Ki}_x} = \frac{2}{\sqrt{\pi}} \sqrt{\mathrm{Fo}_x}, \tag{15}$$

i.e., theoretically we have the one-dimensional case of changes in the relative temperature $\Theta_1^*(0, 0, \tau)/Ki_x$ as a function of Fo_x;

For high values of K_a ($K_a \to \infty$)

$$\lim_{K_a\to \infty} \frac{\Theta_1^*(0,\ 0,\ \tau)}{\mathrm{Ki}_x} = 0, \tag{16}$$

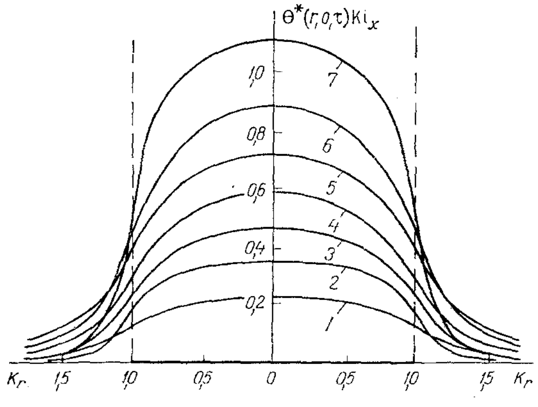

Fig. 2. Dependence of the dimensionless temperature on the relative coordinate $K_r = r/r_0$ for fixed values of Fo_x and $K_a = a_r/a_x$ on the surface (x = 0) of an orthotropic body heated by a constant intensity circular heat source: 1) $Fo_x = 0.1$, $K_a = 10$; 2) 0.1 and 1; 3) 0.2 and 1; 4) 0.4 and 1; 5) 1 and 1; 6) 1 and 0.5; 7) $Fo_x = 1$, $K_a = 0.1$.

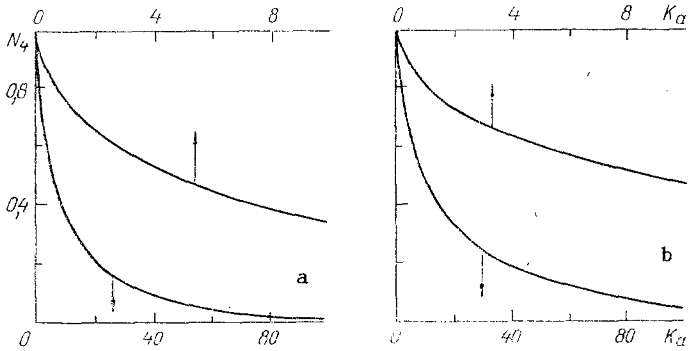

Fig. 3. Dependence of $N_4 = f(K_x, K_a, Fo_r)$ on the relationship K_a: a) for $K_x = 0.1$ and $Fo_r = 0.1$; b) for $K_x = 0.1$ and $Fo_r = 0.2$.

i.e., theoretically we have the two-dimensional limit case when the excess temperature at the central heating point equals zero. In this limit case ($a_r \to \infty$) all the heat delivered to the center will instantaneously be diverted in the radial direction.

The distribution of the dimensionless temperatures $\Theta^*(r, 0, \tau)/Ki_x$ on the surface ($x = 0$) of an orthotropic body heated by a circular heat source is shown in Fig. 2 as a function of the parameters K_f, K_a, and Fo_x. As should have been expected, the absolute level of the values of the temperatures $\Theta^*(r, 0, Fo_x)/Ki_x$ over the section of constant intensity heat source action depends substantially on the relationships of K_a and Fo_x. A low absolute value of the temperature $\Theta^*(r, 0, Fo_x)/Ki_x$ on the heating surface of an orthotropic body by a circular heat source corresponds to small times and large values of K_a while higher absolute levels corresponds to large times and small values of K_a.

Let us examine one of the possible versions of determining the thermophysical characteristics (TFC) resulting from the solution of the problem (2)-(4). Determination of the thermal activity (b_x) of an orthotropic body in the direction of the cylindrical coordinate x can be realized in the initial times of the action (connection) of a low inertial (in thermal respects) circular heater of constant power density q_0. When $\tau \to 0$ (Fo_x, $Fo_r \to 0$), the arguments $K_a^{-1/2}/\sqrt{4Fo_x}$, $1/4Fo_r \to \infty$, here the limit values of the function ierfc(z) will tend to zero as $Z \to \infty$: $\lim \mathrm{ierfc}(Z) \to 0$. It then follows from (12) that the relative excess temperature at the center point ($x = r = 0$) of the circular heater will be described by the expression

$$\Theta_1^*(0,\ 0,\ \tau) = \frac{T_1(0,\ 0,\ \tau) - T_0}{T_0} = \frac{2}{\sqrt{\pi}}\,\mathrm{Lv}_x = \frac{2}{\sqrt{\pi}}\,\mathrm{Ki}_x\sqrt{\mathrm{Fo}_x}, \tag{17}$$

from which

$$b_x = \frac{2q_0\sqrt{\tau}}{\Theta_1(0,\ 0,\ \tau)\sqrt{\pi}}. \tag{18}$$

Determination of the thermal diffusivity coefficient a_r in the direction of the cylindrical coordinate r can be realized by different methods. The first method assumes that the thermal activity b_x [computed in the initial stage of development of the temperature field $\Theta_1(0, 0, \tau)$]

$$\mathrm{ierfc}\left(\frac{1}{2\sqrt{\mathrm{Fo}_r}}\right) = Y, \tag{19}$$

where

$$Y = \frac{1}{\sqrt{\pi}} - \frac{\Theta_1(0,\ 0,\ \tau)}{2q_0\sqrt{\tau}/b_x} = \frac{1}{\sqrt{\pi}} - \frac{\Theta_1^*(0,\ 0,\ \tau)}{2\mathrm{Lv}_x(\tau)}. \tag{20}$$

The appropriate value of the argument $1/(2\sqrt{Fo_r})$ or the number Fo_r directly can be found from this equation by means of known values of Y (for a given time τ). Then

$$a_r = \frac{r_0^2}{\tau}\,\mathrm{Fo}_r. \tag{21}$$

The second method of computing a_r can be realized without knowledge of b_x by the ratio of the temperatures at multiple times.

In order to determine the other thermophysical characteristics b_r, λ_r, a_x, λ_x of the orthotropic body under consideration in the directions of the appropriate cylindrical coordinates r and x, it is sufficient to find the ratios, either $K_b = b_r/b_x$ or $K_a = a_r/a_x$, since the following identities are conserved for this body:

$$K_\lambda = \lambda_r/\lambda_x = K_a = K_b^2, \; K_b = \sqrt{K_a}. \tag{22}$$

The simplest method of determining the relationship K_a is to use the possibility of measuring the temperature $\Theta_1(0, x, \tau)$ on the axis r = 0 at a point removed a distance $x = x_1$ from the heating surface. Then by using the dependence of the temperature $\Theta_1(0, 0, \tau)$ at the center of the heating spot on the surface (r = x = 0) and at a given point $x = x_1$ within the body, the following ratio can be computed:

$$\frac{\Theta_1^*(0, x_1, \tau)}{\Theta_1^*(0, 0, \tau)} = \frac{T_1(0, x_1, \tau) - T_0}{T_1(0, 0, \tau) - T_0} = N_4 = f(K_x, K_a, \mathrm{Fo}_r) = \frac{\mathrm{ierfc}\left(\frac{K_x\sqrt{K_a}}{2\sqrt{\mathrm{Fo}_r}}\right) - \mathrm{ierfc}\left(\frac{\sqrt{1+K_aK_x^2}}{2\sqrt{\mathrm{Fo}_r}}\right)}{1/\sqrt{\pi} - \mathrm{ierfc}\left(\frac{1}{2\sqrt{\mathrm{Fo}_r}}\right)}. \tag{23}$$

Since the computation of the thermal diffusivity a_r raises no difficulties, the numbers $\mathrm{Fo}_r = a_r\tau/r_0^2$ or $\mathrm{Fo}_r(\tau_1) = a_r\tau_1/r_0^2$ will always be known for any time. For a fixed value of $K_x = x/r_0$ as a function of N_4 in (23), only the ratio $K_a = a_r/a_x$ will be an unknown quantity. The ratio N_4 is found from test. Then for specific values of N_4, K_x, $\mathrm{Fo}_r(\tau)$ by using the graphs represented in Fig. 3, the appropriate value of K_a can be found. Computation of the thermal diffusivity a_x is performed by means of the formula

$$a_x = a_r/K_a. \tag{24}$$

The bulk specific heat $c_x\gamma_x = c_r\gamma_r$ and the heat conductivity λ_x and λ_r are computed from equations relating these characteristics when using the identities (22).

If the direction of the heat flux vector $\mathbf{q}(\tau)$ when investigating the TFC of an orthotropic body can vary in the space of the cylindrical coordinates r, x (i.e., local heating of the body under consideration by a circular heat source can occur in experiment in the direction of the axis r), then the thermal activity b_r and the thermal diffusivity a_r can be measured in an analogous manner (exactly the same as in the direction of the coordinate x) without being inserted into the inner space of the orthotropic body.

NOTATION

$\Theta_1(r, x, \tau)$, $\Theta_2(r, x, \tau)$, excess temperatures in the correspondings domains of variation of the variable r (according to the text); r_0, r, x, respectively, the radius of the circle and the cylindrical coordinates; a_r, λ_r, b_r, $c_r\gamma_r$, a_x, λ_x, b_x, $c_x\gamma_x$, thermal diffusivity, the thermal conductivity, and thermal activity, and the bulk specific heat in the r and x coordinate directions; τ, time; $q(\tau)$, q_0, heat flux density; $K_a = a_r/a_x$, $K_\lambda = \lambda_r/\lambda_x$, $K_b = b_r/b_x$, parameters characterizing the relationships between the thermophysical properties in the appropriate directions (r or x); s, p, parameters of the Laplace and Hankel integral transforms; $J_{-1/2}(px)$, Bessel function of half order; $W_{k,\mu}(x)$, Whittaker (degenerate hypergeometric) function; $A_{n,m}$, constant thermal amplitudes; $K_r = r/r_0$, $\Theta^* = \Theta/T_0$; T_0, initial temperature of the anisotropic body; $\mathrm{Ki}_x = q_0r_0/(\lambda_xT_0)$, $\mathrm{Fo}_x = a_z\tau/r_0^2$ and $\mathrm{Ki}_r = q_0r_0/(\lambda_rT_0)$, $\mathrm{Fo}_r = a_r\tau/r_0^2$, Kirpichev and Fourier criteria in the appropriate coordinate directions (x and r); ierfc(x), multiple probability integral; $\mathrm{Lv}_x(\tau) = q_0\sqrt{\tau}/(b_xT_0)$, Lykov criterion [1]; N, an experimentally measurable parameter for the identification of the thermophysical properties of an anisotropic body; $I_0(X)$, $I_1(X)$, $K_0(X)$, $K_1(X)$, modified Bessel functions of corresponding order; $K_x = x/r_0$, a parameter characterizing the relationship between the running x coordinate and the radius r_0 of the heating spot.

LITERATURE CITED

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